Gaussian Process Model Predictive Control of Unmanned Quadrotors

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Outline

Abbreviations

- **GP: Gaussian process**
- **MPC: Model predictive control**
- **NTV: Nonlinear time varying**
- **CG: Conjugate gradient**
- **MSE: Mean square errors**

Problem Formulation

Figure: Quadrotor in AUT Lab

Problem Formulation

Quadrotor State-Space Functions:

$$\boldsymbol{\xi}_{k+1} = \mathcal{F}_{\xi}(\boldsymbol{\xi}_k, \mathbf{u}_{\xi,k}) \tag{1a}$$

$$\boldsymbol{\eta}_{k+1} = \mathcal{F}_{\eta}(\boldsymbol{\eta}_k, \mathbf{u}_{\eta,k}) \tag{1b}$$

- $\boldsymbol{\xi} = [x, \dot{x}, y, \dot{y}, z, \dot{z}]^T$, $\mathbf{u}_{\xi} = [U_1, u_x, u_y]^T$,
  $\boldsymbol{\eta} = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$, $\mathbf{u}_{\xi} = [U_2, U_2, U_3]^T$,
- $\mathcal{F}_{\xi}$ and $\mathcal{F}_{\eta}$ are time-varying nonlinear functions

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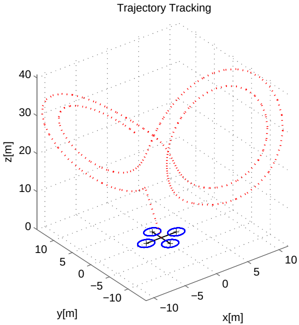
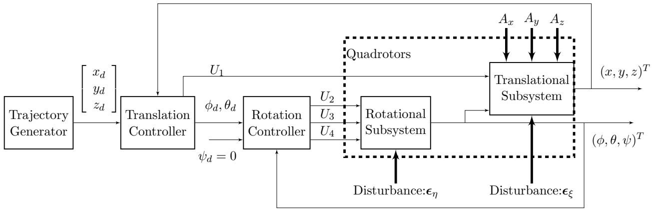


Proposed Control Scheme



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GP based MPC

Numerical Simulation

Conclusion

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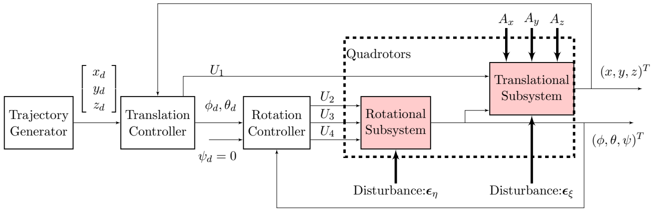
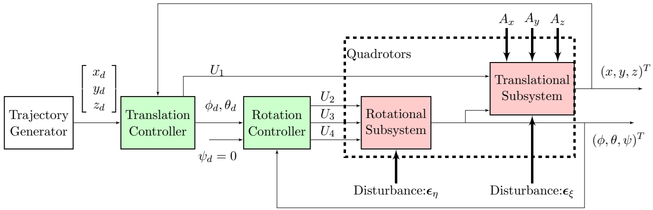


Contributions:

- Translational subsystem
- Rotational subsystem

} GP Modelling

Proposed Control Scheme



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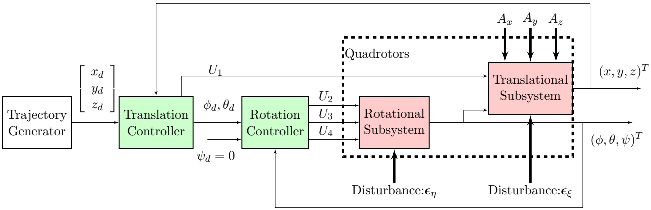

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- Gradient based solution to GPMPC

Quadrotor Modelling using GP

General NTV Form:

$$\mathbf{x}_{k+1} = \mathcal{F}(\mathbf{x}_k, \mathbf{u}_k) \tag{2}$$

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Gaussian Process Models:

- Probabilistic data-driven model;
- Specifying a squared exponential covariance function $\mathbf{K}(\mathbf{x}_{k,i}, \mathbf{x}_{k,j})$;
  $\mathbf{K}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{x}}_j) = \sigma_s^2 \exp(-\frac{1}{2}(\tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j)^T \boldsymbol{\Lambda}(\tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j)) + \sigma_n^2$

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- GP Inputs $\tilde{\mathbf{x}}_k = [\mathbf{x}_k, \mathbf{u}_k]^T$;
  GP Outputs $\delta\mathbf{x}_{k+1} = \mathbf{x}_{k+1} - \mathbf{x}_k$;

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  GP Outputs $\delta\mathbf{x}_{k+1} = \mathbf{x}_{k+1} - \mathbf{x}_k$;
- Hyperparameters learning: Minimize the negative log of likelihood (CG);

GP based MPC

$$\underbrace{\mathbf{V}_k^*(\mathbf{x}_k, \mathbf{r}_k, \mathbf{u}_{k-1})}_{\text{deterministic NMPC}}$$

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Implementation Issues:

- Uncertainty propagation → GP predictions at uncertain inputs;

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Implementation Issues:

- Uncertainty propagation → GP predictions at uncertain inputs;
- Effective Solutions → Gradient based optimization;

Gradient based optimization

Unconstrained optimization question:

$$\mathbf{U}_k^* = \arg\min_{\mathbf{U}_k^*} \mathbb{E}\left\{\mathbf{V}_k^*\right\} \tag{3a}$$

$$\text{s.t. } \boldsymbol{\mu}_{k+1} = \mathcal{G}(\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k, \mathbf{u}_k) \tag{3b}$$

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- Gradient based linear search strategy

$$\mathbf{U}_{k+1} = \mathbf{U}_k + \boldsymbol{\alpha} \cdot \underbrace{\frac{\partial \mathbb{E}\{\mathbf{V}_k\}}{\partial \mathbf{U}_k}}_{\text{Gradient}} \tag{4}$$

$$\frac{\partial \mathbb{E}\{\mathbf{V}_k\}}{\partial \mathbf{U}_k} = \frac{\partial \mathbb{E}\{\mathbf{V}_k\}}{\partial \boldsymbol{\mu}_{k+1}} \frac{\partial \boldsymbol{\mu}_{k+1}}{\partial \mathbf{U}_k} + \frac{\partial \mathbb{E}\{\mathbf{V}_k\}}{\partial \boldsymbol{\Sigma}_{k+1}} \frac{\partial \boldsymbol{\Sigma}_{k+1}}{\partial \mathbf{U}_k} + \frac{\partial \mathbb{E}\{\mathbf{V}_k\}}{\partial \mathbf{U}_k} \tag{5}$$

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- End when $\mathbb{E}\{\mathbf{V}_k\}_{\mathbf{U}_k} \leq \epsilon$;

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- End when $\mathbb{E}\{\mathbf{V}_k\}_{\mathbf{U}_k} \leq \epsilon$;
- Multi-Start with different initial values;

Numerical Simulation

$$\boldsymbol{\xi}_{k+1} = \mathcal{F}_{\xi}(\boldsymbol{\xi}_k, \mathbf{u}_{\xi,k}; \boldsymbol{\theta}_{\xi}) + \boldsymbol{\epsilon}_{\xi} \tag{6a}$$

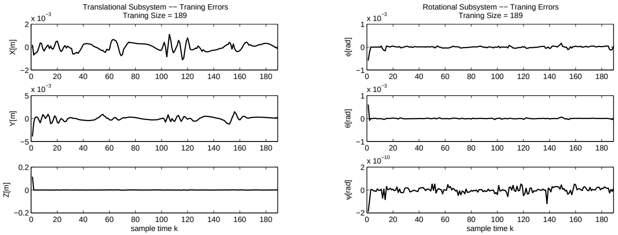
$$\boldsymbol{\eta}_{k+1} = \mathcal{F}_{\eta}(\boldsymbol{\eta}_k, \mathbf{u}_{\eta,k}; \boldsymbol{\theta}_{\eta}) + \boldsymbol{\epsilon}_{\eta} \tag{6b}$$

- Specify the parameters $\boldsymbol{\theta}_{\xi}$ and $\boldsymbol{\theta}_{\eta}$
- Specify two Gaussian noises $\boldsymbol{\epsilon}_{\xi}$ and $\boldsymbol{\epsilon}_{\eta}$
- Observations are generated by using a NMPC
- All observations are used to train the GP model
- 50 repeats
- Compare to the Nelder-Mead method (derivative-free)

Numerical Simulation

Modelling Results

	Training MSE
Translational Subsystem	1.8693×10^{-7}
Rotational Subsystem	8.5238×10^{-9}



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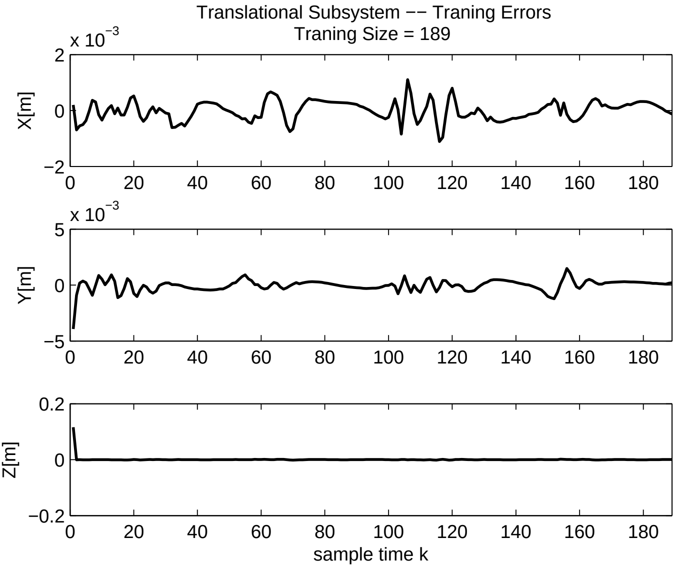

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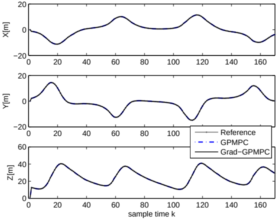
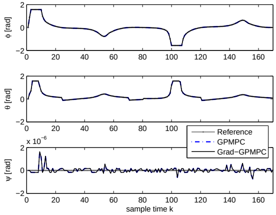
Control Results

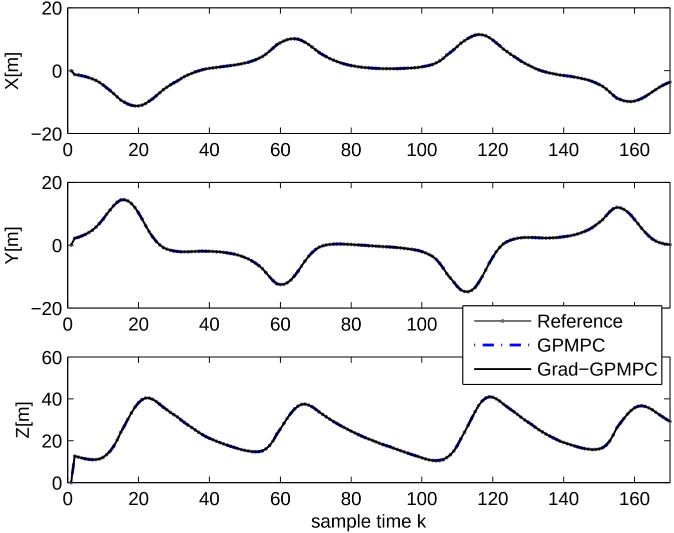
- "Grad-GPMPC" only requires approximately **30%** time of "GPMPC" to solve the MPC problem

Positions	Tracking MSE	Attitudes	Tracking MSE
X	3.3418×10^{-4}	ϕ	1.5743×10^{-8}
Y	5.1399×10^{-5}	θ	5.5213×10^{-9}
Z	0.0010	ψ	6.4365×10^{-14}



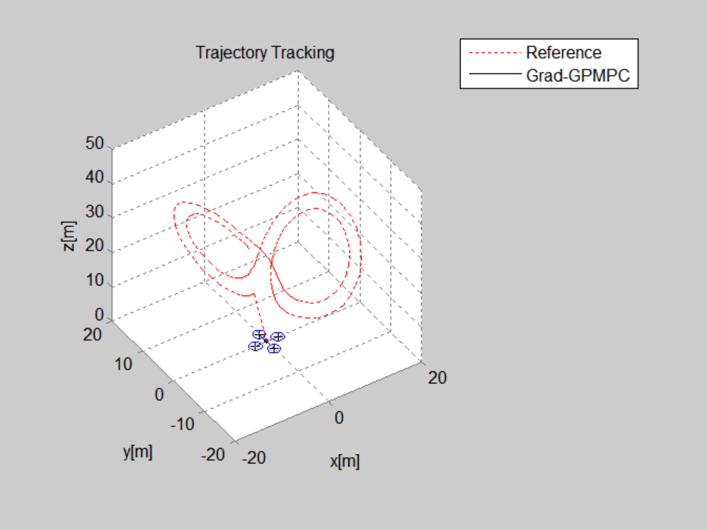


Numerical Simulation



Tracking Results

Viewing angle: az=-52, el=14



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- Proposed GPMPC performs well in Quadrotor tracking problems
- Proposed gradient based method solve the GPMPC problem more efficiently

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Future Works:

- GPMPC to control Quadrotor with constraints
- Guaranteed stability and robustness against uncertainties

Thanks!

Questions?