PSO for Convolved GP Models

Particle Swarm Optimization for Convolved Gaussian Process Models

2014 IEEE World Congress on Computaional Intelligence (IJCNN Program)

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May 9, 2016

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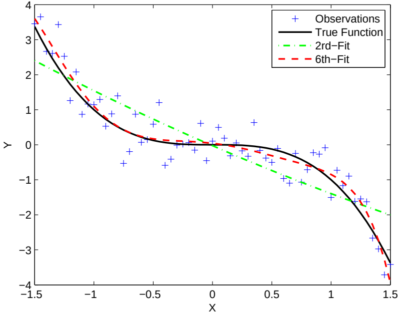
UNIVERSITY OF NEW ZEALAND

Outline

Introduction

Learning from Data



A comparison of different model for learning question.
Original model is $y = x^3$ and corrupted by a noise $\epsilon \sim \mathcal{N}(0, 0.5)$.

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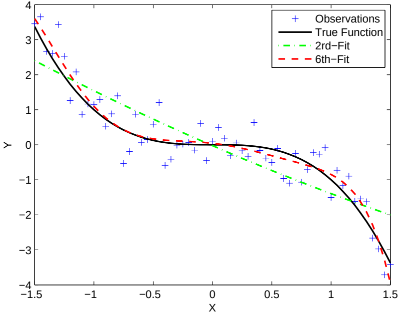

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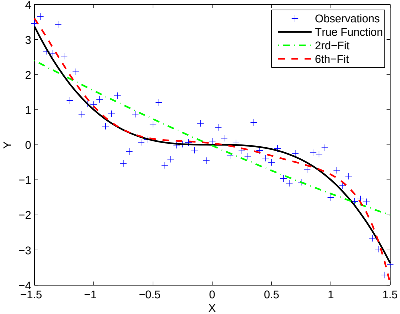

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Questions?

- Best Model?

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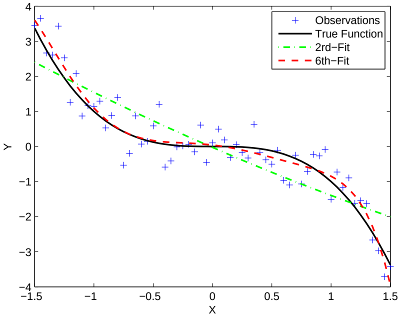

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Original model is $y = x^3$ and corrupted by a noise $\epsilon \sim \mathcal{N}(0, 0.5)$.

Questions?

- Best Model?
- Errors? and Measurements?

Introduction

Bayesian Model

- Use prior
- Consider all possible models

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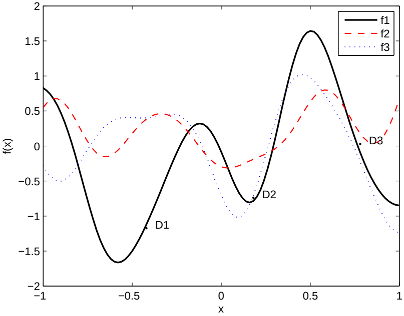

Bayesian Regression Technique
Contributions: $f1 > f3 > f2$

Introduction

Bayesian Model

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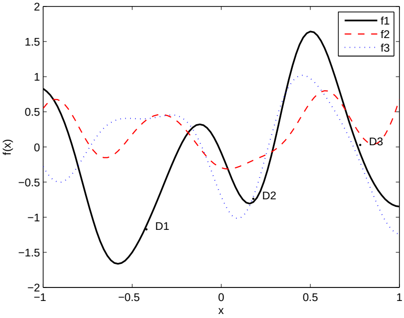

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Questions?

- Defining Prior

Introduction

Bayesian Model

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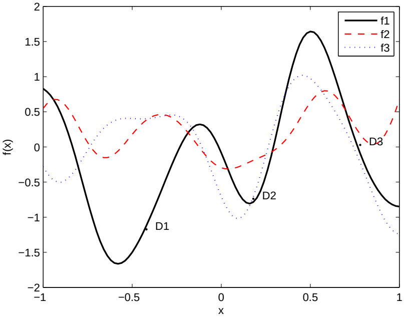


Bayesian Regression Technique
Contributions: $f1 > f3 > f2$

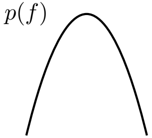
Questions?

- Defining Prior
- Analytically intractable

Gaussian Process Models

- Standard GP
- Multi-output GP
- Model Learning
- CGP Learning using PSO

GP Model – Standard GP



Gaussian distribution

$f \sim \mathcal{N}(\mu, \sigma^2)$

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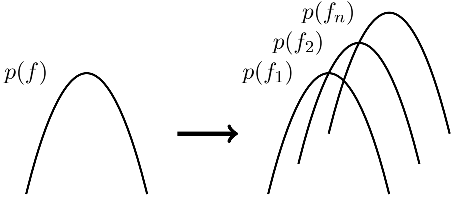

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$\mathbf{f} \sim \mathcal{GP}(\boldsymbol{\mu}(\cdot), \mathbf{cov}(\cdot))$

- GP: A collection of random variables with a joint Gaussian distribution among any finite number of them

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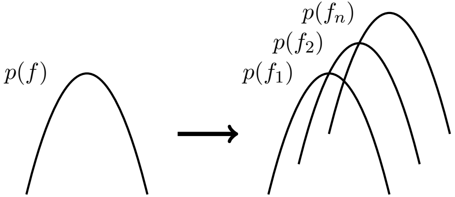

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Gaussian processes
$\mathbf{f} \sim \mathcal{GP}(\boldsymbol{\mu}(\cdot), \mathbf{cov}(\cdot))$

- GP: A collection of random variables with a joint Gaussian distribution among any finite number of them
- Fully specified by mean and variance functions

$$\begin{aligned}\mu(\mathbf{x}^*) &= \mathbf{K}_{\mathbf{f}^*,\mathbf{f}}(\mathbf{K}_{\mathbf{f},\mathbf{f}} + \sigma^2\mathbf{I})^{-1}\mathbf{y} \\ var(\mathbf{x}^*) &= \mathbf{K}_{\mathbf{f}^*,\mathbf{f}^*} + \sigma^2 - \mathbf{K}_{\mathbf{f}^*,\mathbf{f}}(\mathbf{K}_{\mathbf{f},\mathbf{f}} + \sigma^2\mathbf{I})^{-1}\mathbf{K}_{\mathbf{f},\mathbf{f}^*}.\end{aligned} \tag{1}$$

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Question: MIMO problem using GP model?

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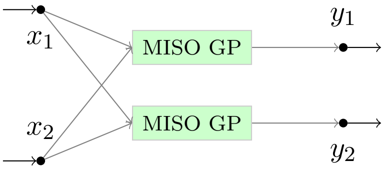

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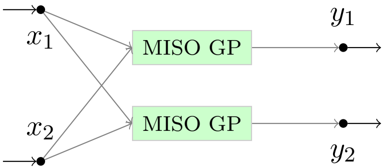

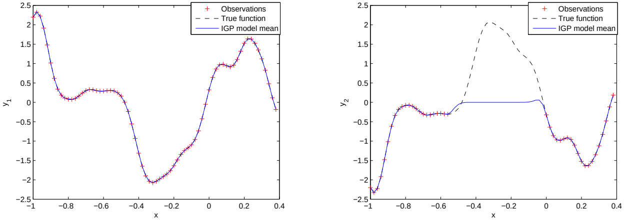

Modelling using IGP model

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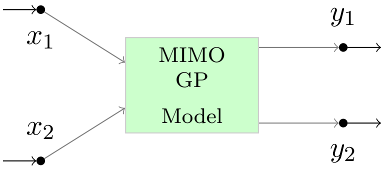

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Convolved Gaussian process models:

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Convolved Gaussian process models:

- Construct a new covariance function

$$\begin{bmatrix} \mathbf{K_{11}} & \cdots & \mathbf{K_{1n}} \\ \mathbf{K_{21}} & \cdots & \mathbf{K_{2n}} \\ \vdots & \vdots & \vdots \\ \mathbf{K_{n1}} & \cdots & \mathbf{K_{nn}} \end{bmatrix} \tag{2}$$

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  - input-output correlations
  - correlations between outputs
- No additional computations of inference and predictions.

PSO for Convolved GP Models

GP Model – Model Learning

The log-likelihood function:

$$\mathbf{L}\left(\theta\right) = -\frac{1}{2}\mathbf{y}^{\mathbf{T}}\ \mathbf{K}_{\mathbf{y},\mathbf{y}}^{-\mathbf{1}}\ \mathbf{y} - \frac{1}{2}\mathbf{log}\left|\ \mathbf{K}_{\mathbf{y},\mathbf{y}}\ \right| - \frac{\mathbf{MJ}}{2}\log\mathbf{2}\pi \quad (3)$$

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	Search Range [0,100]		Search Range [0,1]	
	Fitness 1	Fitness 2	Fitness 1	Fitness 2
NLL	≈ 51	≈ 46	≈ 242269	≈ 212314

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	Search Range [0,100]		Search Range [0,1]	
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NLL	≈ 51	≈ 46	≈ 242269	≈ 212314
MSE	0.5313	0.2199	0.0101	0.0032

GP Model – CGP learning using PSO

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- Explore CGP learning using PSO

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Works in this paper:

- Explore CGP learning using PSO
- Use MSE as fitness;

Simulation Results

- Simulation Design
- Simulation Results

Simlation Results – Design

Numerical Example:

- MISO Modelling Problem

$$\begin{aligned} y(k) &= 0.893y(k-1) + 0.037y^2(k-1) \\ &\quad -0.05y(k-2) + 0.157u(k-1) \\ &\quad -0.05u(k-1)y(k-1) \end{aligned} \tag{4}$$

- $y_1 = y$, $y_2 = -y_1$ or $y_2 = exp(y_1)$
- Generate 1000 observations
- Each simulation repeats 10 times

Experimental Parameters:

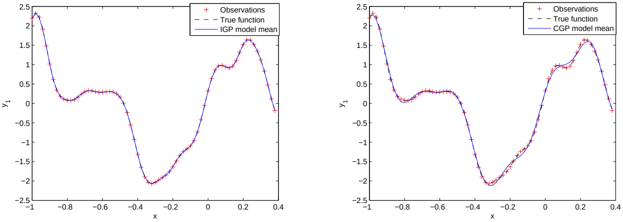
Symbol	Description	Quantity
$\mathbf{T}_{\max}$	Maximum Iterations	2000
c_1,c_2	Acceleration Factors	1.49445
ω_{start}	Start Inertial Factor	0.4
ω_{end}	End Inertial Factor	0.9
k	Shape Control Factor	0.8
CG Restarts	Restart Times	25×2000
$\|\Delta\xi\|$	Minimum Fitness Variation	$1e-5$
$\nu_{d,i}, \upsilon_q$	Coefficients Search Range	$[1, 2]$
α_i, β_j	$\mathbf{P_d}$, $\mathbf{P_q}$Elements Search Range	$[0, 1]$

Simlation Results – Results

Simulation 1 – Modelling with Missing Data

Simlation Results – Results

Simulation 1 – Modelling with Missing Data

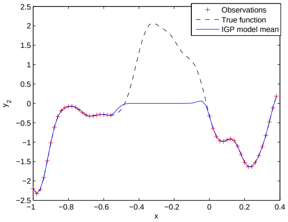


(c) y_1 – IGP with full data

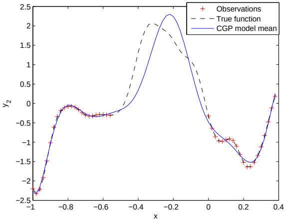
(d) y_1 – CGP with full data

Simlation Results – Results

Simulation 1 – Modelling with Missing Data



(e) y_2 – IGP with miss data



(f) y_2 – CGP with miss data

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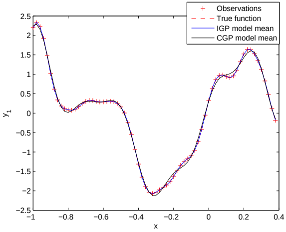
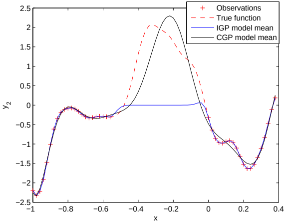


(g) y_1 with full data



(h) y_2 with miss data

- Similar performance with full data

Simlation Results – Results

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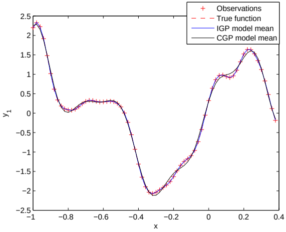
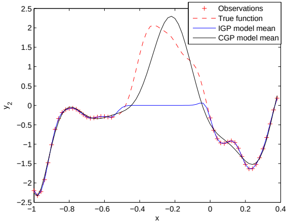


(i) y_1 with full data

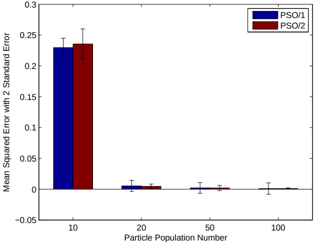


(j) y_2 with miss data

- Similar performance with full data
- CGP has better performance with miss data

Simlation Results – Results

Simulation 2 – MISO System Modelling



Results of MISO modelling problem.
PSO/2 denotes PSO with MSE fitness, while PSO/2 is PSO with LL fitness.

Simlation Results – Results

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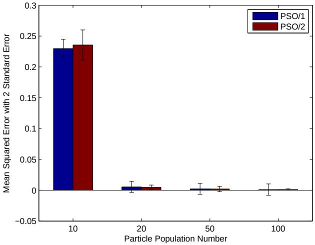

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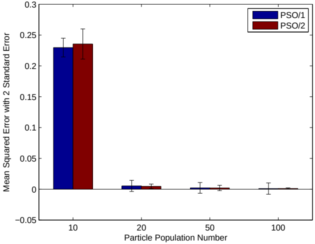

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- 2 PSOs have similar performance
- Population number 25 is used in the simulations

Simlation Results – Results

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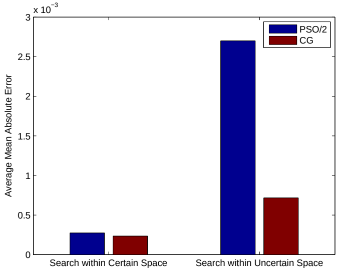

Statistic Results of MISO system modelling.
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Simlation Results – Results

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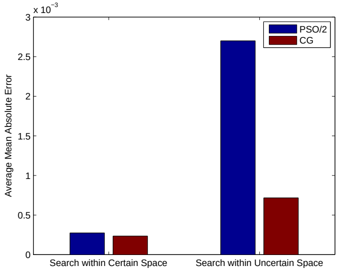

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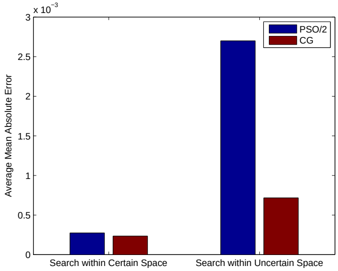

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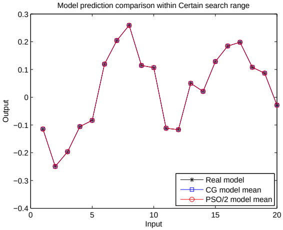
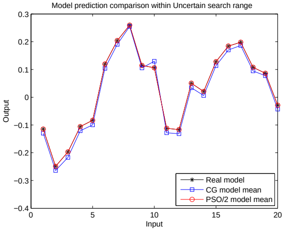


(k) Certain Search range



(l) Uncertain Search range

Model prediction comparison
within certain or uncertain search ranges.

- Similar performance when search space is well-defined
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Simlation Results – Results

Simulation 4 – MIMO System Modelling

- Two outputs are correlated
- Linear and nonlinear correlations

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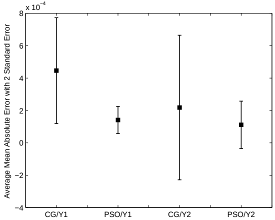
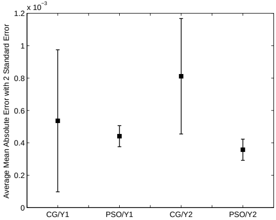


(m) $y_2 = -y_1$



(n) $y_2 = \mathbf{exp}(y_1)$

Simlation Results – Results

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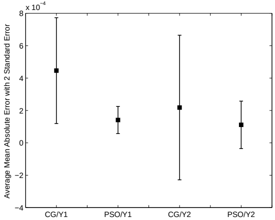

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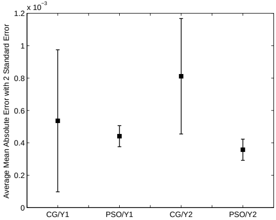

(p) $y_2 = \mathbf{exp}(y_1)$

- Model with PSO produces smaller average MSE and SE within a uncertain search range

Simlation Results – Results

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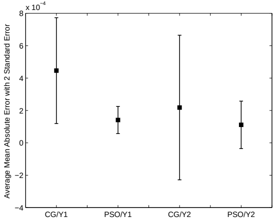

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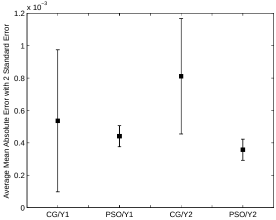

(r) $y_2 = \mathbf{exp}(y_1)$

- Model with PSO produces smaller average MSE and SE within a uncertain search range
- CGP model with PSO performs well both for linear and nonlinear problems

Conclusion

- Better modelling ability for missing-data problems

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- Clear indications of MSE than LL fitness

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- Clear indications of MSE than LL fitness
- Better learning ability for uncertain search range
- Work well for linear and nonlinear systems

Future Works

- More complex questions:NLTI to NLTV

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- Use improved PSO methods

Thanks!

Questions?